

Unparticle as a field with continuously distributed mass

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February 1, 2008

Abstract

We point out that the notion of an unparticle, recently introduced by Georgi, can be interpreted as a particular case of a field with continuously distributed mass considered in ref.[14]. We also point out that the simplest renormalizable extension of the $SU_c(3) \otimes SU_L(2) \otimes U(1)$ Standard Model is the extension with the replacement of the $U(1)$ gauge propagator $\frac{1}{k^2} \rightarrow \frac{1}{k^2} + \int_0^\infty \frac{\rho(t)}{-t+k^2+i\epsilon} dt$ with $\int_0^\infty \rho(t) dt < \infty$.

Recently, Georgi [1] made an interesting observation that a nontrivial scale invariant sector of scale dimension d_U might manifest itself at low energy as a nonintegral number d_U of invisible massless particles, dubbed unparticle U with untrivial phenomenological implications.¹

In this note we point out that the notion of an unparticle, introduced by Georgi can be interpreted as a particular case of a field with continuously distributed mass [14]. We also point out that the simplest renormalizable extension of the $SU_c(3) \otimes SU_L(2) \otimes U(1)$ Standard Model is the extension with the replacement of the $U(1)$ gauge propagator $\frac{1}{k^2} \rightarrow \frac{1}{k^2} + \int_0^\infty \frac{\rho(t)}{-t+k^2+i\epsilon} dt$ with $\int_0^\infty \rho(t) dt < \infty$.

Let us start with N scalar fields [14] $\phi_k(x)$ with masses m_k ($k = 1, 2, \dots, N$). For the field $\Phi_{int}(x, m_k, c_k, N) = \sum_{k=1}^N c_k \phi_k(x)$ free propagator has the form

$$D_{int}(k^2, m_k, c_k, N) = \sum_{k=1}^N \frac{|c_k|^2}{(k^2 - m^2 + i\epsilon)} = \int_0^\infty \frac{\rho(t, c_k, m_k, N)}{k^2 - t + i\epsilon} dt, \quad (1)$$

where the spectral density is $\rho(t, c_k, m_k, N) = \sum_{k=1}^N |c_k|^2 \delta(t - m_k^2)$. In the limit $k \rightarrow \infty$ $\rho(t, c_k, m_k, N) \rightarrow \rho(t)$ and the propagator $D_{int}(k^2, m_k, c_k, N) \rightarrow D_{int}(k^2) = \int_0^\infty \frac{\rho(t)}{k^2 - t + i\epsilon} dt$ [14]. For instance, for $m_k^2 = m_0^2 + \frac{k}{N} \Delta^2$ and $|c_k|^2 = \frac{1}{N}$ we find that the limiting spectral density is $\rho(t) = \frac{1}{\Delta^2} \theta(t - m^2) \theta(m^2 + \Delta^2 - t)$. For the limiting spectral density $\rho(t) \sim t^{\delta-1}$ we find that the propagator $D_{int}(k^2) \sim (k^2)^{\delta-1}$ that corresponds to the case of unparticle propagator. In other words, for the limiting spectral density $\rho(t) \sim t^{\delta-1}$ the field $\phi_{int}(x, \rho(t)) = \lim_{N \rightarrow \infty} \Phi_{int}(x, m_k, c_k, N)$ can be interpreted as unparticle.²

It should be stressed that the limiting field $\phi_{int}(x, \rho(t))$ can be interpreted as a field describing scalar particle with continuously distributed mass [14]. Moreover we believe it is important to consider possible experimental consequences for arbitrary spectral density $\rho(t)$ but not only for $\rho(t) \sim t^{-\delta}$ corresponding to unparticle case. One can introduce the selfinteraction Lagrangian of the field $\phi_{int}(x, \rho(t))$ in standard way as

$$L_{int}(\phi_{int}(x, \rho(t))) = -\lambda(\phi_{int}(x, \rho(t)))^4 \quad (2)$$

¹See also some implications in Refs. [2]- [13].

²The interpretation of the unparticle as a tower of massive particles was also proposed in ref.[15]

For finite $\int_0^\infty \rho(t)dt$ the asymptotics of propagator $D_{int}(k^2) \sim \frac{1}{k^2}$ and the model (2) is renormalizable one. It should be noted that for Georgi noninteracting scalar unparticle effective Lagrangian is

$$L_{unp} = \frac{1}{2} \partial_\mu \phi (-\partial^\mu \partial_\mu)^{-\delta} \partial^\mu \phi. \quad (3)$$

The Lagrangian $L_{tot} = L_{int} + L_{unp}$ has generalized scale invariance [16] and all ultra-violet divergent integrals can be made finite by subtraction of infinities at zero external momentum.

For the Standard Model based on $SU_c(3) \otimes SU_L(2) \otimes U(1)$ gauge group there are several ways to generalize it by the introduction of the fields with continuously distributed mass. Namely, it is possible to introduce new scalar field $\phi_{int}(x, \rho(t))$ with continuously distributed mass and introduce the interaction with standard Higgs doublet field $H(x)$

$$L_{int}(\phi_{int}(x, \rho(t)), H(x)) = -\lambda_2(\phi_{int}(x, \rho(t)))H^+(x)H(x). \quad (4)$$

After electroweak symmetry breaking the singlet field $\phi_{int}(x, \rho(t))$ will mix with the standard Higgs boson that can change drastically [15] Standard Model predictions for the Higgs boson search at LHC.

The generalization to the case of vector fields is the following. Consider the Lagrangian

$$L_0 = \sum_{k=1}^N \left[-\frac{1}{4e_k^2} F^{\mu\nu,k} F_{\mu\nu,k} + \frac{m_k^2}{2e_k^2} (A_{\mu,k} - \partial_\nu \phi_k)^2 \right], \quad (5)$$

where $F_{\mu\nu,k} = \partial_\mu A_{\nu,k} - \partial_\nu A_{\mu,k}$. The Lagrangian (5) is invariant under gauge transformations

$$A_{\mu,k} \rightarrow A_{\mu,k} + \partial_\mu \alpha_k, \quad (6)$$

$$\phi_k \rightarrow \phi_k + \alpha_k. \quad (7)$$

For the field $B_\mu = \sum_{k=1}^N A_{\mu,k}$ free propagator in transverse gauge is

$$D_{\mu\nu}(p) = (g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}) \left(\sum_{k=1}^N \left(\frac{e_k^2}{p^2 - m_k^2} \right) \right). \quad (8)$$

In the limit $N \rightarrow \infty$

$$D_{\mu\nu}(p) \rightarrow (g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}) D_{int}(p^2), \quad (9)$$

where

$$D_{int}(p^2) = \int_0^\infty \frac{\rho(t)}{p^2 - t + i\epsilon} dt \quad (10)$$

and $\rho(t) \geq 0$. One can introduce the interaction of the field B_μ with fermion field ψ in standard way, namely

$$L_{int} = \bar{\psi} \gamma_\mu \psi B^\mu. \quad (11)$$

The simplest generalization of the Standard Model consists in the replacement of the $U(1)$ gauge field propagator

$$(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}) \frac{1}{p^2} \rightarrow (g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}) D_{int}(p^2). \quad (12)$$

This generalization of the Standard Model preserves the renormalizability for finite $\int_0^\infty \rho(t) dt$ because the ultraviolet asymptotics of $D_{int}(k^2)$ coincides with free propagator. For $\rho(t) \sim t^{\delta-1}$ we reproduce the case of vector unparticle. For the propagator $D_{int}(p^2) = g_1^2 (\frac{1}{p^2} + \frac{1}{(p^2 - M^2)})$ we obtain generalization of the Standard Model with single additional vector field. Current TEVATRON experimental bound on M is $M \geq 850 \text{ GeV}$ [18]. For the model with arbitrary $D_{int}(p^2)$ experimental bound will depend on the spectral density $\rho(t)$.

It should be stressed that the fields with continuously distributed mass arise naturally in n-dimensional field theories [17]. Consider five-dimensional scalar field with the Lagrangian

$$L_5 = \frac{1}{2} (\partial_\mu \phi \partial^\mu \phi - \phi f(-\partial_4^2) \phi), \quad (13)$$

where $\mu = 0, 1, 2, 3$. For the Lagrangian L_5 free propagator is

$$D_0 = \frac{1}{k_\mu k^\mu - f(k_4^2)}. \quad (14)$$

For the field $\phi(x_\mu, x_4 = 0)$ propagator is proportional to $\int_{-\infty}^\infty \frac{dk_5}{k_\mu k^\mu - f(k_5^2)}$ that corresponds to the case of the field with continuously distributed mass. The fact that unparticle can be interpreted as a result of the compactification of 5-dimensional space-time for a model with AdS_5 metric was mentioned also in ref.[15]. The difference of our model and the model proposed in [15] is that we use 5-dimensional Lagrangian (13) which explicitly

violates five-dimensional Poincare group. The Lagrangian (13) is invariant only under 4-dimensional Poincare group.

This work was supported by the Grant RFBR 07-02-00256, I am indebted to a referee for pointing out to me a reference [15] and critical remarks.

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